P H Y S I C A L   R E V I E W   L E T T E R S

Generation of Large-Amplitude Coherent-State Superposition via Ancilla-Assisted Photon Subtraction

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We propose and demonstrate a novel method to generate a large-amplitude coherent-state superposition (CSS) via ancilla-assisted photon subtraction. The ancillary mode induces quantum interference of indistinguishable processes in an extended space, widening the controllability of quantum superposition at the conditional output. We demonstrate this by a simple time-separated two-photon subtraction from continuous wave squeezed light. We observe the largest CSS of traveling light ever reported without correcting any imperfections, which will enable various quantum information applications with CSS states.

Superposition of macroscopically distinct coherent states, often called coherent-state superposition (CSS), is regarded as a realization of Schrödinger's famous cat paradox. Typical CSS states are defined as $|C_\pm\rangle = \frac{1}{\sqrt{N_\pm}}(|\alpha\rangle \pm |-\alpha\rangle)$, where $|\pm\alpha\rangle$ are coherent states with amplitudes $\pm\alpha$. The $|C_+\rangle$ ($|C_-\rangle$) is called an even- (odd-) CSS state and $|\alpha|^2$ is often regarded as its "size" since it reflects the distance of two superposed coherent states. Such states have been realized in a few physical systems [1,2]. Among them large size CSS states of travelling light are important for many quantum information tasks such as linear-optics quantum computation [3,4], quantum teleportation [5], and quantum metrology [6]. In practice, one can generate approximate $|C_\pm\rangle$ by subtracting photons from a squeezed vacuum [7]. Along this line, single-photon subtraction has been demonstrated, generating odd catlike states [8–10]. But their sizes were still small, typically $|\alpha|^2 \simeq 1.0$.

Although subtracting more than two photons leads to larger states, experiments get more challenging due to rapid decrease of the success probability. Recently an alternative way utilizing a photon-number state and homodyne detection was proposed and a generation of the state $\sqrt{2/3}|2\rangle - \sqrt{1/3}|0\rangle$ from a pulsed two-photon state was demonstrated [11]. It is close to a superposition of two squeezed states where they are displaced from the origin in opposite directions by the amount $\simeq 1.2$ in the phase space. If 3.5 dB squeezing is further applied, the state would be $|C_+\rangle$ with $|\alpha|^2 \simeq 2.6$. However, it remains a challenge to apply squeezing onto such a squeezed CSS state.

In this Letter, we propose and demonstrate a novel way to enhance the size of CSS states without resorting to further subtraction of more than two photons or squeezing operations. Instead we introduce an ancillary mode to assist in suppressing the weights of smaller number photons in a CSS state, and hence to enhance its size. In the

experiment, we implement it in the time domain, by time-separated two-photon subtraction from continuous wave (cw) squeezed light.

Photon subtraction is done by tapping a small fraction of the squeezed vacuum for photon counting, and by selecting the transmitted state conditioned on the detection of photons. A two-photon subtraction from a single-mode squeezed vacuum (without ancillae) is described as

$$\hat{a}^2 \hat{S}(\epsilon_0)|0\rangle = \beta_0 \hat{S}(\epsilon_0)(\epsilon_0 \hat{a}^{\dagger 2} + 1)|0\rangle, \qquad (1)$$

where $\hat{a}$ is an annihilation operator describing one-photon subtraction, $\hat{S}$ is a squeezing operator, ϵ_0 represents the degree of squeezing [12] and $\beta_0 = \epsilon_0/(1 - \epsilon_0^2)$. This is a squeezed state of the superposition $\sqrt{2}\epsilon_0|2\rangle + |0\rangle$, which well approximates an even CSS state with the size up to $|\alpha|^2 \simeq 1.0$ [7]. However, it is still possible to further increase the size if one could optimize the ratio between $|0\rangle$ and $|2\rangle$ in the superposition, independently from ϵ_0 [13].

Suppose two independent photon subtraction events instantaneously occur at time t_1 and t_2, respectively, on a cw squeezed vacuum generated from an optical parametric oscillator (OPO) as depicted in Fig. 1(a). Since the cw squeezed vacuum has a finite bandwidth ζ_0, the effect of each photon subtraction spreads over the temporal wave packet $\psi(t - t_{1,2}) = \sqrt{\zeta_0} e^{-\zeta_0|t-t_{1,2}|}$ in the transmitted beam [14–16]. When the time separation $\Delta \equiv |t_2 - t_1|$ satisfies $\zeta_0\Delta \lesssim 1$, these two packets are highly overlapped and two temporal waveforms defined as $\Psi_\pm(t) \equiv [\psi(t - t_1) \pm \psi(t - t_2)]/\sqrt{2(1 \pm I_\Delta)}$ with $I_\Delta \equiv (1 + \zeta_0\Delta)e^{-\zeta_0\Delta}$ appear as orthonormal mode functions to describe the process [13,16]. The symmetric mode $\Psi_+(t)$ acts as the main mode to be measured while the asymmetric mode $\Psi_-(t)$ serves as an ancilla and the essential mechanism can be well described by these two modes. Precisely speaking, subtracted photons have finite correlations with the other
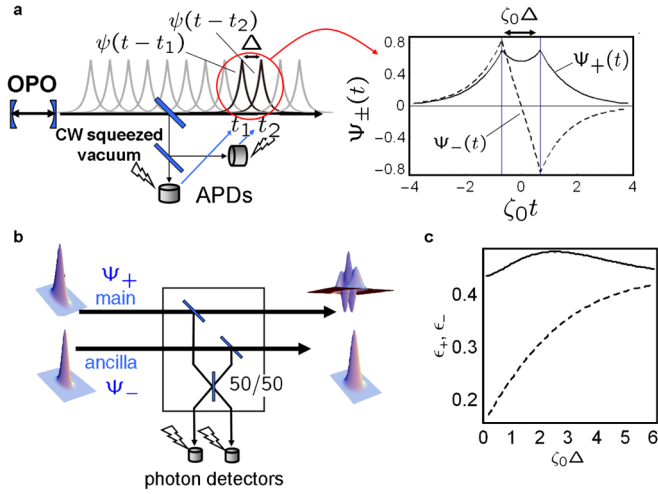
FIG. 1 (color online). (a) The time-separated two-photon subtraction and the temporal mode functions $\Psi_\pm(t)$. OPO: optical parametric oscillator, APD: avalanche photo diode. (b) Schematic of the ancilla-assisted photon subtraction. (c) Δ dependence of the squeezing ϵ_+ (solid line) and ϵ_- (dashed line). $\epsilon = 0.3$.

temporal modes in the transmitted beam, but the effects of those modes can simply be taken as a small optical loss [13]. Then the temporal two-mode model can be translated into a simplified spatial two-mode model with modes $\Psi_\pm$ and a 50/50 beam splitter (BS) as illustrated in Fig. 1(b). The coincidence of clicks at each photon detector always means that two photons must come from either Ψ_+ or Ψ_-, and not that single photon from each mode. This is due to the bunching nature of photons. These two subtraction processes are indistinguishable, producing a superposition,

$$(\hat{a}_+^2 - \hat{a}_-^2)\hat{S}_+(\epsilon_+)\hat{S}_-(\epsilon_+)|0\rangle_+|0\rangle_-, \qquad (2)$$

where the subscripts $\pm$ indicate modes $\Psi_\pm$. The $\epsilon_\pm$ represent effective degrees of squeezing in each mode which are determined by a fully multimode theory and depend on Δ, ζ_0, and the OPO pumping parameter ϵ [13]. Figure 1(c) shows typical behaviors of $\epsilon_\pm$ as functions of Δ. Since ϵ_- is relatively small at $\zeta_0\Delta \lesssim 1$, using Eq. (1) and (2) can be

rewritten as follows by neglecting terms proportional to ϵ_-^2 or higher orders,

$$\text{Eq. (2)} \approx \beta_+ \hat{S}_+(\epsilon_+)\left(\epsilon_+ \hat{a}_+^{\dagger 2} + \left(1 - \frac{\epsilon_-}{\beta_+}\right)\right)|0\rangle_+ \hat{S}_-(\epsilon_-)|0\rangle_-$$

$$\equiv \mathcal{N}|\Phi\rangle_+ \hat{S}_-(\epsilon_-)|0\rangle_-, \qquad (3)$$

where $\beta_+ = \epsilon_+/(1 - \epsilon_+^2)$, a normalized state $|\Phi\rangle$ and a normalization factor $\mathcal{N}$ are introduced. Compared with Eq. (1), it can be seen that the ancilla mode provides additional controllability for tuning the superposition in the main mode through ϵ_-, which is simply controlled by changing the time separation Δ as shown in Fig. 1(c). This allows us to coherently modify the photon-number distribution of the state in the main mode and, in particular, suppress the weight of small photon-number components to increase the size of the CSS state. Note that, in a rigorous cw model, the state in the main mode is slightly degraded to a statistical mixture $\hat{\rho}_+ = (1 - C_0)|\Phi\rangle\langle\Phi| + C_0\hat{S}_+|0\rangle\langle0|\hat{S}_+^\dagger$ even without practical imperfections due to a weak entanglement between the main and the ancilla mode. Figure 2 compares the theoretical Wigner functions of $\hat{\rho}_+$ based on the full cw model [13] for $\zeta_0\Delta = 0$ and 1.4, i.e., without and with the ancilla assistance. These correspond to the CSS states of $|\alpha|^2 = 1.2$ ($\zeta_0\Delta = 0$) and 2.6 ($\zeta_0\Delta = 1.4$) with the fidelities of 0.928 and 0.932, respectively, where the latter Δ is chosen to maximize the size of CSS while preserving a high fidelity (>0.9). For larger Δ, C_0 becomes non-negligible as discussed later.

A schematic of our experimental setup is shown in Fig. 3. It is built on our previous single-photon subtraction setup [10] with qualitative updates to cope with long data acquisition for hours. A cw Ti:sapphire laser of 860 nm is used as a light source. A squeezed beam is generated from an OPO which employs a periodically poled KTiOPO$_4$ (PPKTP) as nonlinear crystal. The OPO is pumped by a frequency doubled beam of 20 mW from a second harmonic generation (SHG) cavity with a KNbO$_3$ crystal. This corresponds to $\epsilon = 0.3$. The OPO bandwidth is $\zeta_0/2\pi \sim 4.5$ MHz (HWHM). A small fraction (10%) of the beam is tapped by a tapping BS (TBS) and going through two
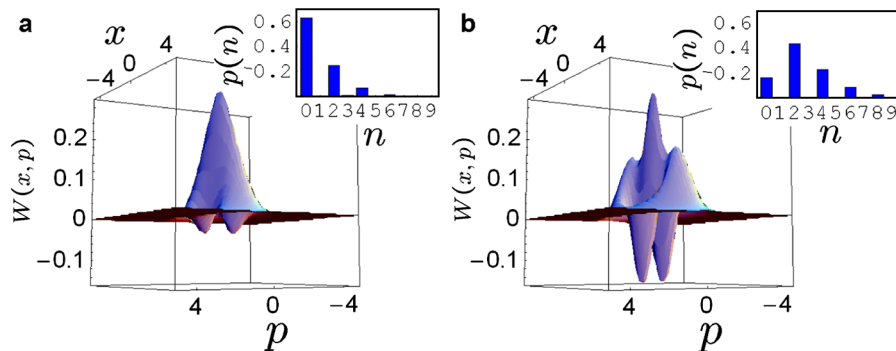

FIG. 2 (color online). Theoretical Wigner functions of the CSS state generated by the time-separated two-photon subtraction and their photon-number distributions. $\epsilon = 0.3$. (a) $\zeta_0\Delta = 0$ and (b) $\zeta_0\Delta = 1.4$ correspond to approximate $|C_+\rangle$ of $|\alpha|^2$ of 1.2 and 2.6, respectively.
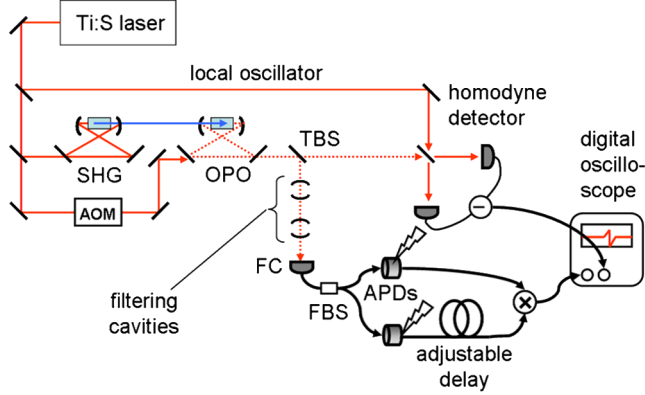
FIG. 3 (color online). Experimental setup of the time-separated two-photon subtraction. SHG: second harmonic generation cavity, OPO: optical parametric oscillator, AOM: acousto-optic modulator, FC: fiber coupler, TBS: tapping beam splitter ($R = 10\%$) FBS: 50/50 fiber beam splitter, and APDs: Si avalanche photo diodes (Perkin-Elmer SPCM-AQR-14).

filtering Fabry-Perot cavities of 2 mm-long and of 0.9 mm-long, respectively. The unwanted photons in nondegenerate modes extending over a wide range of frequencies are well filtered out by these cavities and only the component in the degenerate mode around 860 nm is guided into two avalanche photo diodes (APDs) [9,10].

The beam transmitted through the TBS is continuously measured by a homodyne detector. All cavities are actively locked on the resonance for 860 nm by a weak coherent beam and electronic feedback. The coherent beam is periodically chopped by an acousto-optic modulator (AOM), which defines distinct time bins by presence and absence of the coherent beam, and thus enables the alternate sequence of locking and photon counting without stray light. Also via phase locking the coherent beam to parametric gain of the OPO, we obtain phase information of the measured state from interference between the coherent beam and the local oscillator (LO) beam at the homodyne detector.

Clicks of APDs announce that a conditional state appears in the transmitted beam. The time separation Δ is adjusted by an electronic delay line. The pumping of $\epsilon = 0.3$ at the OPO outputs a 3.0 dB squeezed vacuum in temporal mode $\Psi_+(t)$ and the simultaneous-click rate of the two APDs is about 6 counts/s for $\Delta = 0$ ns and less than 1 count per second for $\Delta \sim 100$ ns. The conditional state in the main mode is measured by a homodyne detector to reconstruct its Wigner function. For every pair of clicks of the APDs with desired Δ, a continuous photocurrent of the homodyne detector is stored. A quadrature amplitude in the main mode is extracted from the continuous data by applying the numerical mode filter with the shape of $\Psi_+(t)$ which is equivalent to an optical LO with the waveform of $\Psi_+(t)$ [9,10].

Experimentally reconstructed Wigner functions and photon-number distributions of $\hat{\rho}_+$ with $\Delta = 0$ and 32 ns are given in the upper row of Fig. 4(a). The reconstruction

was performed by the conventional maximum likelihood estimation method [17] from 22 000 points of quadrature amplitudes in the raw experimental data. No experimental imperfections are corrected in the reconstruction procedure, whereas most of the previous works corrected the losses at the state evaluation step [8,9,11]. Figure 4(a) clearly shows that with finite Δ, the two positive peaks corresponding to the components $|\pm\alpha\rangle$ become apart, i.e., the size-increase of α.

Ideally, the two-photon subtracted states of $\Delta = 0$ and 32 ns ($\zeta_0\Delta = 0$ and 0.9, respectively) would approximate $|C_+\rangle$ of $|\alpha|^2 = 1.2$ and 2.0, respectively, with high fidelities ($>90\%$). In the experiment, the fidelities of 76% and 60%, for $|\alpha|^2 = 1.2$ and 2.0, respectively, are achieved. A particular feature of the macroscopic quantum superposition is the presence of negative dips in the Wigner function due to the interference between two distinct wave packets. For $\Delta = 32$ ns, the negative dips are clearly observed at the sides of the origin. This is, to our knowledge, the largest CSS state of the travelling light ever observed, showing the negative Wigner function without any compensation of experimental imperfections. Such quality is achieved by use of a highly pure squeezed vacuum generated from our low loss OPO [10]. The sources of imperfection are separately investigated. The OPO has 0.4% intracavity loss. Associated with 9.5% transmittance of the coupling mirror, it gives 0.95 of the escape efficiency. The propagation efficiency from the OPO to the homodyne detector is 0.95. The quantum efficiency of the homodyne detector and its electronic noise give effective detection efficiency 0.98. The spatial overlap between the squeezed vacuum and the LO gives another efficiency of 0.96. In total, the overall efficiency is estimated around 0.85. Theoretical Wigner functions including these imperfections are shown in the lower row of Fig. 4(a). The experimental results are almost in agreement with the theoretical models [18] while the negativity is slightly degraded by some uncleared imperfections. We also note that the maximum CSS state is obtained around $\Delta = 48$ ns [see Fig. 2(b)] while the experimentally observed negative dips are limited up to 32 ns since the low count rates at large Δ also degrades the negativity.

The photon-number distribution in Fig. 4(a) also reveals a mechanism of the ancilla-assisted photon subtraction. Suppression of the small photon-number components by increasing Δ is clearly observed. Our result should also be contrasted to the squeezed state superposition $\sqrt{2/3}|2\rangle - \sqrt{1/3}|0\rangle$ generated in [11]. In our case, the even-number distribution extends at least up to $n = 4$, and is clearly deviated from that of the squeezed vacuum, which should be another signature of the macroscopic nature of the superposition.

Figure 4(b) plots the average photon numbers $\langle\hat{n}_\pm\rangle$ of modes $\Psi_\pm$ as a function of Δ, that agree with the theory well indicated by lines [18]. The increase of $\langle\hat{n}_+\rangle$ in the
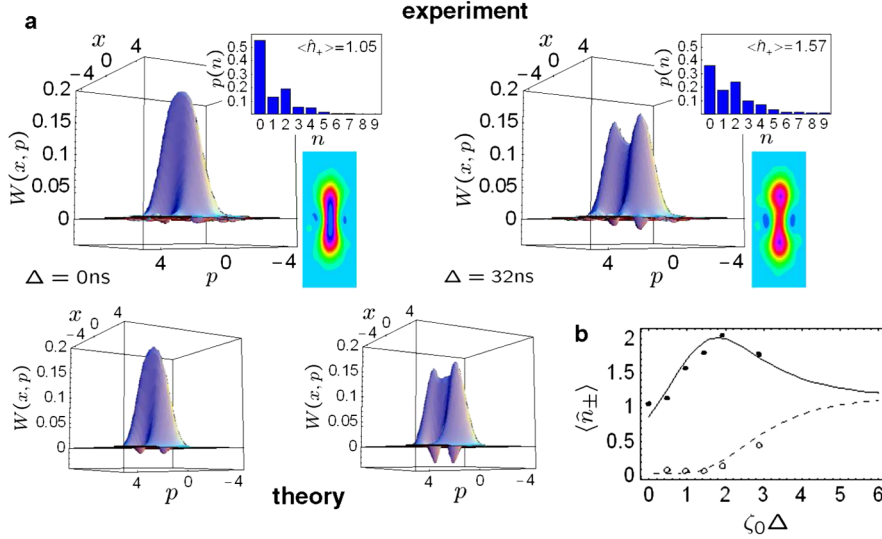
FIG. 4 (color online). (a) Wigner functions of the state $\hat{\rho}_+$ in mode Ψ_+ for $\Delta = 0$ ns ($\zeta_0 \Delta = 0$, left) and $\Delta = 32$ ns ($\zeta_0 \Delta = 0.90$, right). Upper row: experimental Wigner functions ($W(x, p)$), their contours, and photon-number distributions ($p(n)$). Lower row: model calculation of the Wigner functions. (b) Average photon numbers in modes $\Psi_\pm$. Upper (filled) plots: experimental $\langle \hat{n}_+ \rangle$ with $\Delta = 0$, 32, 48, 64, 96 ns, from left to right. Lower (unfilled) plots: experimental $\langle \hat{n}_- \rangle$ with $\Delta = 32$, 48, 64, 96 ns, from left to right. Solid ($\langle \hat{n}_+ \rangle$) and dashed ($\langle \hat{n}_- \rangle$) lines are the theoretical predictions including 15% optical loss.

range $\zeta_0 \Delta \lesssim 2$ supports an experimental evidence of the size-increase. In this range most of the photons accumulate in the main mode with fewer photons in the ancillary mode. As Δ increases beyond $\zeta_0 \Delta \gtrsim 2$, however, $\langle \hat{n}_- \rangle$ starts to increase, and photons in modes Ψ_+ and Ψ_- are getting entangled. It induces degradation of the purity of the reduced state in the main mode. For $\zeta_0 \Delta \gg 1$, the state in the main mode is degraded to a completely mixed state $\hat{\rho}_+ \rightarrow \frac{1}{2} \hat{S}_+ (|2\rangle\langle 2| + |0\rangle\langle 0|) \hat{S}_+^\dagger$.

Finally, we point out that the above drawback of the purity degradation could be overcome if one would take advantage of an ancillary mode which is not entangled to the main mode. This is done by preparing a coherent state $|\alpha\rangle$ in the ancillary mode. The improved scheme is represented in terms of generic two modes A and B as

$$(\hat{a}_A^2 - \hat{a}_B^2) \hat{S}_A |0\rangle_A |\alpha\rangle_B = (\hat{a}_A^2 - \alpha^2) \hat{S}_A |0\rangle_A |\alpha\rangle_B. \quad (4)$$

Since the coherent state is an eigenstate of $\hat{a}$, one can make a superposition of two- and zero-photon subtractions in mode A without entangling two modes, i.e., without degradation. Here the coherent amplitude simply plays the role of Δ in the time-separated two-photon subtraction. In practice, the experiment will get more involved since one has to lock the relative phase of these two modes. Also a related scheme was proposed recently [19].

In conclusion, we have proposed a concept of the ancilla-assisted photon subtraction and experimentally demonstrated it by the time-separated two-photon subtraction from cw squeezed light. Because of the quantum interference assisted by the ancillary mode, we have successfully observed and characterized the largest CSS state of traveling light so far showing the negative dips of the Wigner function without any corrections of measurement imperfections. Our experimental scheme involves the trade-off between the size enlargement and the degradation of the state purity. We pointed out that the trade-off can be

circumvented by extending it to the one with a coherent-state ancilla.

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